Baulkham Hills High School

**2022** Year 12 Trial Examination

# Mathematics Advanced

**General Instructions**

- Reading time - 10 minutes
- Working time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-35, show relevant mathematical reasoning and/or calculations

**Total marks: 100**

**Section I - 10 marks** (pages 4-6)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

**Section II - 90 marks** (pages 7-29)

- Attempt Questions 11-35
- Allow about 2 hours and 45 minutes for this section

# Section I

**10 marks**
**Attempt Questions 1-10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

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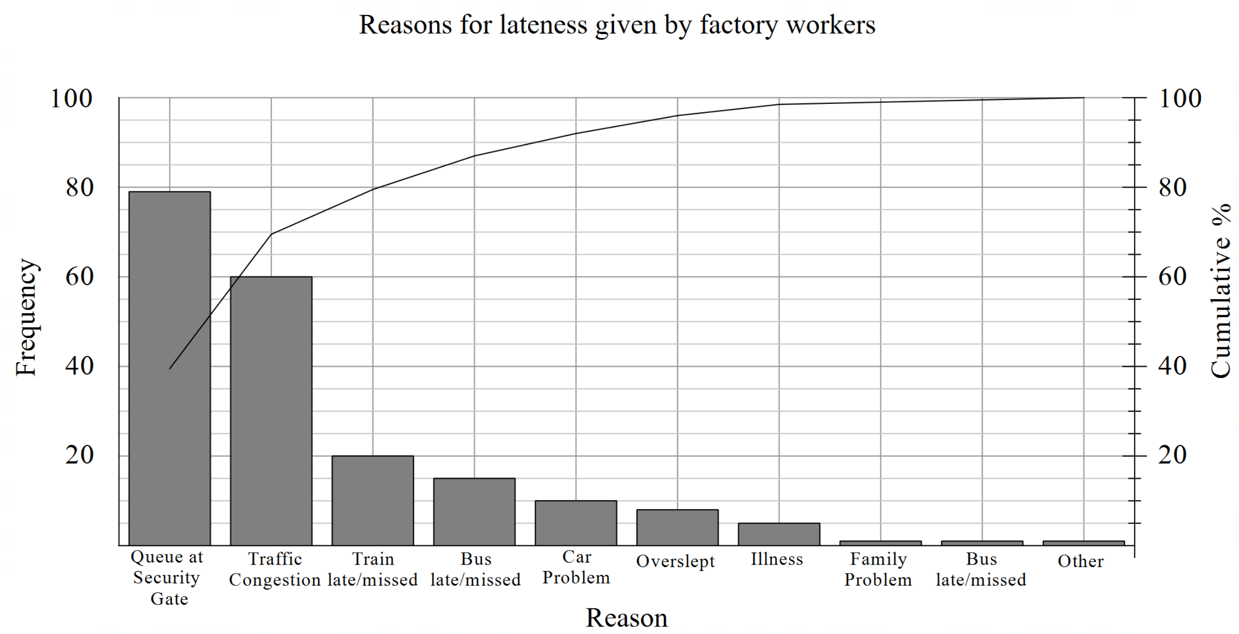
**1** In a survey to find the main causes of lateness in a factory's work force, a random sample of 200 employees who were late for work were asked the reason why. The Pareto chart below shows the results.




What is the minimum number of causes that accounts for the top 80% of the problems?

A. 1
B. 2
C. 3
D. 4

**2** The weights of a particular bug species are normally distributed with mean 700g and standard deviation 20g. What percentage of bugs have a mass exceeding 740g?

A. 2.5%
B. 5%
C. 16%
D. 34%

**3** If $\cos A = \dfrac{3}{5}$ and $0° < A < 90°$, which of the following represents the value of $\cot A$?

A. $\dfrac{3}{5}$

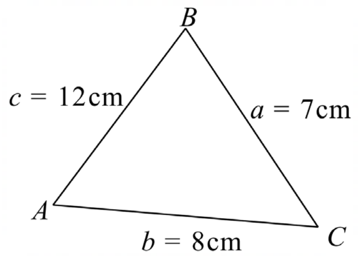
B. $\dfrac{3}{4}$

C. $\dfrac{4}{5}$

D. $\dfrac{4}{3}$

**4** In $\Delta ABC$, $a = 7\text{cm}$, $b = 8\text{cm}$ and $c = 12\text{cm}$. What is the size of angle $A$ to the nearest minute?

A. 34°5'
B. 34°6'
C. 39°50'
D. 106°4'




**5** If three positive numbers are inserted between 2 and 1250 such that the resulting sequence is a geometric progression, which of the following is not among the numbers inserted?

A. 10
B. 25
C. 50
D. 250

**6** The table below shows a discrete probability distribution for a random variable $X$.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X = x)$ | $2b$ | $b$ | $4b$ | $5b$ |

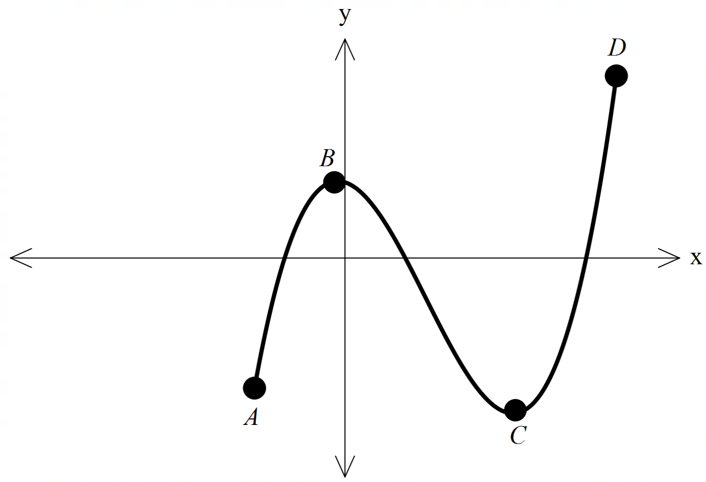
What is the expected value of $X$?

A. $\dfrac{1}{12}$
B. 1
C. 3
D. 36

**7** Which of the following is the domain for $y=\dfrac{1}{x-4}$?

A. $(-\infty,4)\cup(4,\infty)$
B. $(-4,4)$
C. $(-\infty,4]\cup[4,\infty)$
D. $(-\infty,\infty)$

**8** Given the graph below, which of the following points on this curve shows the local maximum?




A. A
B. B
C. C
D. D

**9** The function $y=\log_{10}x$ is transformed to $y=3\log_{10}[2(x+5)]-4$ using only four steps. Which of the following is not one of the correct steps to transform $y=\log_{10}x$ to the new graph?

A. Translate down by 4 units
B. Translate to the left by 5 units
C. Vertical dilation by a factor of 3
D. Horizontal dilation by a factor of 2

**10** Use the *z*-score table provided at the back of the paper to answer this question. A random variable *X* is normally distributed, $P(X\le 22)=61.79\%$ and $P(X\le 17)=35.94\%$. Which of the following is the mean?

A. 1.89
B. 7.58
C. 18.74
D. 19.73

**End of Section I**

# Section II

**90 marks**
**Attempt Questions 11-35**
**Allow about 2 hours and 45 minutes for this section**
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

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**Question 11** (2 marks)

Simplify $2+\frac{t^2}{t+1}-\left(1+\frac{1}{t+1}\right)$. **2**

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**Question 12** (1 mark)

Prove $\tan\theta(1-\cot^2\theta)+\cot\theta(1-\tan^2\theta)=0$. **1**

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**Question 13** (3 marks)

The table below shows the future value of an annuity with a contribution of $1.

| Table of future value interest factors | | | | | |
|---|---|---|---|---|---|
| *Period* | *Interest rate per period* | | | | |
| | 1% | 2% | 3% | 4% | 5% |
| 3 | 3.0301 | 3.0604 | 3.0909 | 3.1216 | 3.1525 |
| 4 | 4.0604 | 4.1216 | 4.1836 | 4.2465 | 4.3101 |
| 5 | 5.1010 | 5.2040 | 5.3091 | 5.4163 | 5.5256 |
| 6 | 6.1520 | 6.3081 | 6.4684 | 6.6330 | 6.8019 |

a) Angela is saving for a holiday by contributing $700 into an annuity that pays interest at the rate of 8% p.a., compounded half-yearly. Use the table above to find how much she will have in 3 years' time. **1**

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b) Grace is saving for the same trip and needs $6000 in total. How much more than Angela does she need to contribute each half year, using the same annuity, if she wishes to have enough money in 3 years' time? **2**

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**Question 14** (2 marks)

Find the second derivative of $y = \ln(4x - 1)$. **2**

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**Question 15** (2 marks)

Differentiate $y = x^2 e^{6x}$. **2**

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**Question 16** (2 marks)

Differentiate $y = \frac{\sin 5x}{x^4}$. **2**

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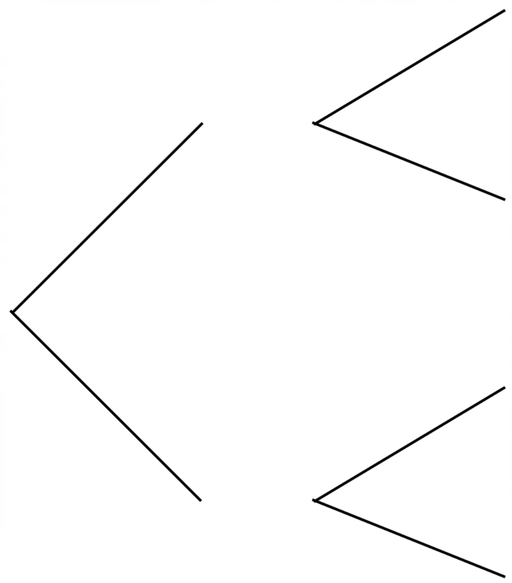
**Question 17** (4 marks)

In a raffle 50 tickets are sold, and there is one first prize and one second prize. Jody buys 15 tickets in the raffle. The first prize is drawn then the second prize.

a) Complete the probability tree diagram below, showing all outcomes. **2**



b) What is the probability that Jody wins the first prize, but not the second prize? **1**

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c) Find the probability that Jody wins at least one prize? **1**

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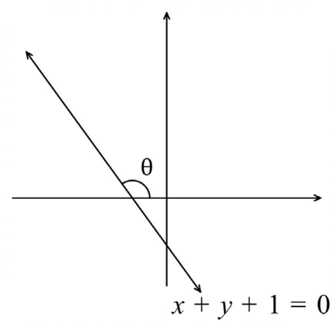
**Question 18** (2 marks)

Determine the angle of inclination $\theta$ of the line $x + y + 1 = 0$. **2**

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**Question 19** (3 marks)

Differentiate $f(x) = 5x^2 - 3x$ from first principles. **3**

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**Question 20** (2 marks)

Find the area bounded by the curve $y = 3e^{2x}$, the $x$-axis and the lines $x = 1$ and $x = 5$. **2**

Leave your answer correct to 1 decimal place.

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**Question 21** (2 marks)

Solve the equation $\log_5(x+1) - \log_5 x = 2$. **2**

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**Question 22** (3 marks)

Evaluate the definite integral $\int_{\frac{\pi}{4}}^{\pi} \tan^2 x \, dx$, leaving your answer in exact form. **3**

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**Question 23** (2 marks)

Evaluate $\int_1^3 \frac{2\,dx}{5x-2}$ correct to 2 decimal places. **2**

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**Question 24** (3 marks)

Fill in the following table and use the trapezoidal rule with 4 function values to approximate $\int_4^7 2^x\,dx$. **3**

| $x$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $y$ | 16 | | | |

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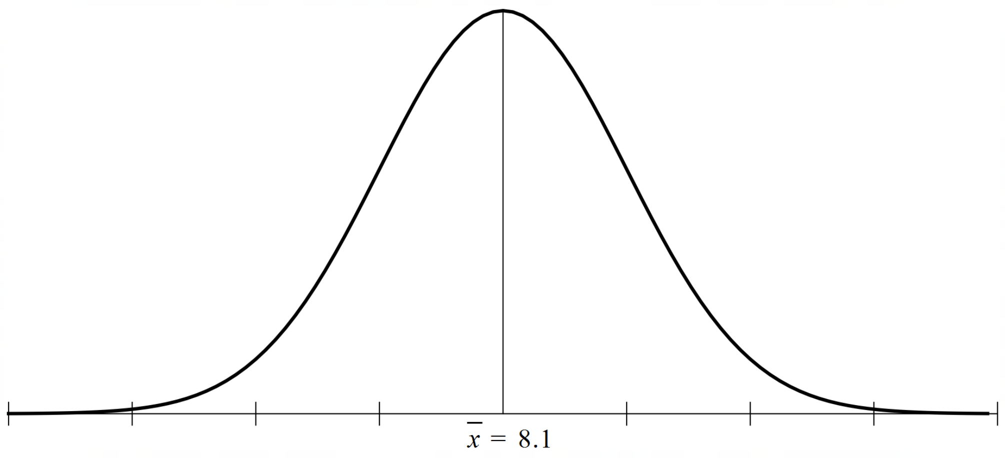
**Question 25** (3 marks)

Find the equation of the normal to the curve $y = x\cos x$ at the point where $x = \dfrac{\pi}{2}$. **3**

**Question 26** (4 marks)

The weights of metal drums are normally distributed with a mean of 8.1kg and a standard deviation of 2kg.




a) What weight would have a $z$-score of $-2$? **1**

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b) Use the $z$-score table provided at the back of the paper to answer this question. **3**
Find $P(8.4 \leq X \leq 11)$, where $X$ is a random variable, representing the weight of the drums?

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**Question 27** (8 marks)

Consider $f(x) = x^3 - 3x^2 + 3$ in the domain $-1 \le x < 3$.

a) Find the stationary point(s) and determine their nature. **3**

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b) Find the point(s) of inflection. **2**

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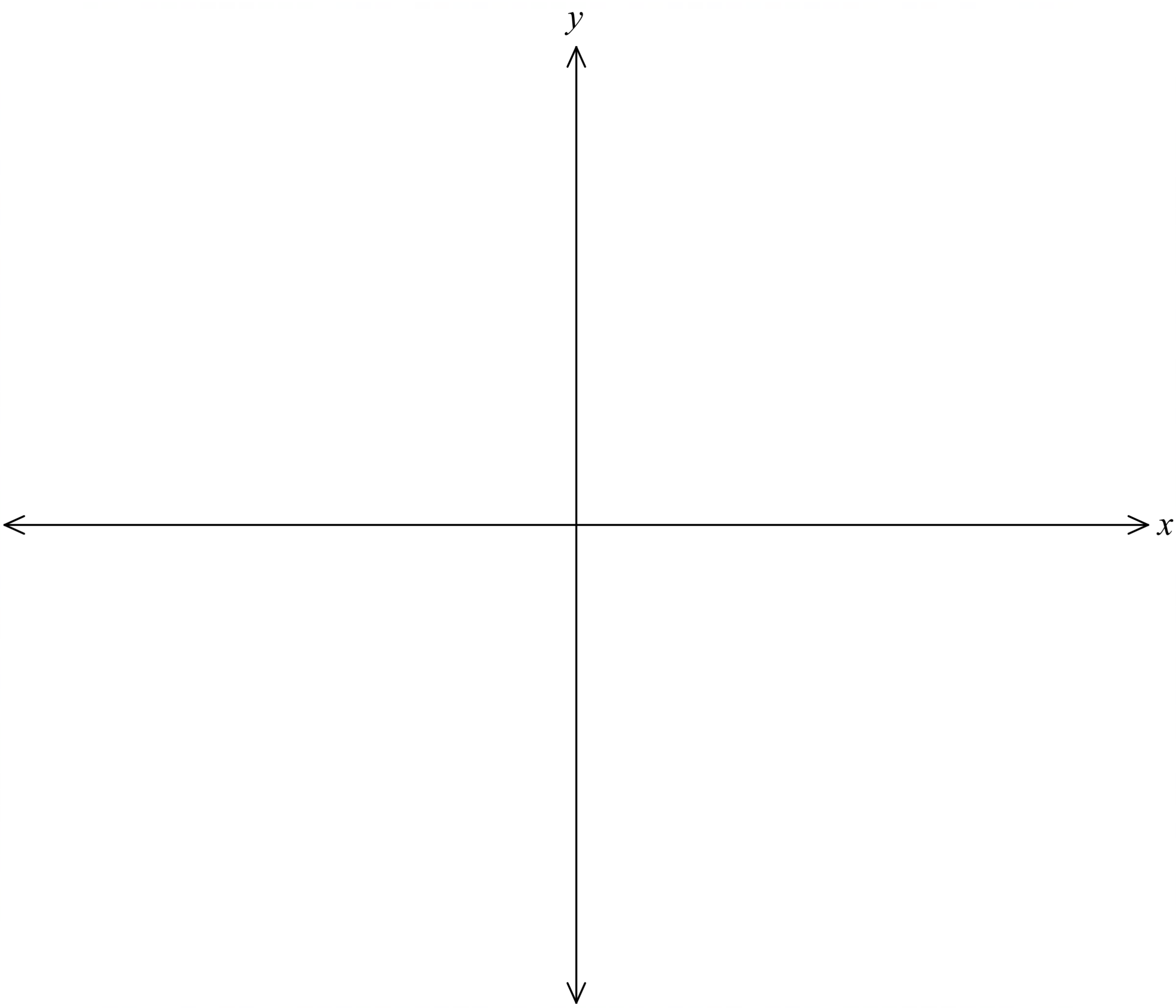
**Question 27 continues onto the next page**

c) Sketch the graph of $y = f(x)$ showing all features. Do not find the $x$ −intercepts. **3**

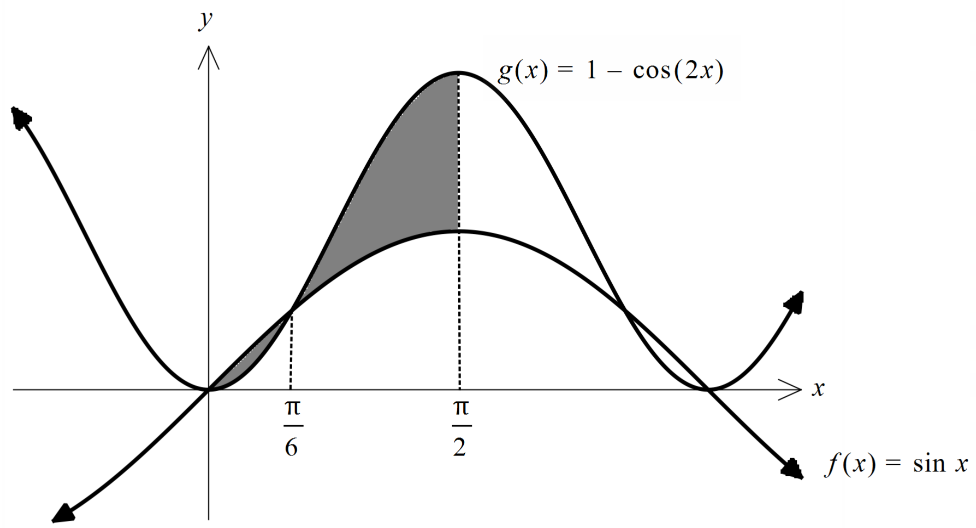


**Question 28** (3 marks)

Solve $4^x - 2^{x+2} = 32$. **3**

**Question 29** (3 marks)

The diagram below shows two curves $f(x) = \sin x$ and $g(x) = 1 - \cos(2x)$.




Find the area of the shaded region correct to 2 decimal places. **3**

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**Question 30** (4 marks)

A continuous random variable, $X$, has the following probability density function.

$$f(x) = \begin{cases} 3^x & \text{for } 0 \le x \le k \\ 0 & \text{for all other values of } x \end{cases}$$

a) Find the value of $k$ correct to 2 decimal places. **2**

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b) Evaluate the $8^{\text{th}}$ decile correct to 2 decimal places. **2**

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**Question 31** (3 marks)

Given $f''(t) = t + \sqrt{t}$, find the equation of the curve $f(t)$, given $f(1) = 1$ and $f'(1) = 2$. **3**

**Question 32** (12 marks)

A particle moves in a straight line so that its distance $x$ metres from the origin is given by

$x = 4\cos\left(t - \frac{\pi}{3}\right)$ for $0 \le t \le \pi$.

a) Where is the particle initially? **1**

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b) When does the particle first come to rest? **2**

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c) When does the particle first pass through the origin? **1**

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d) Find when the rate of change of velocity is zero. **2**

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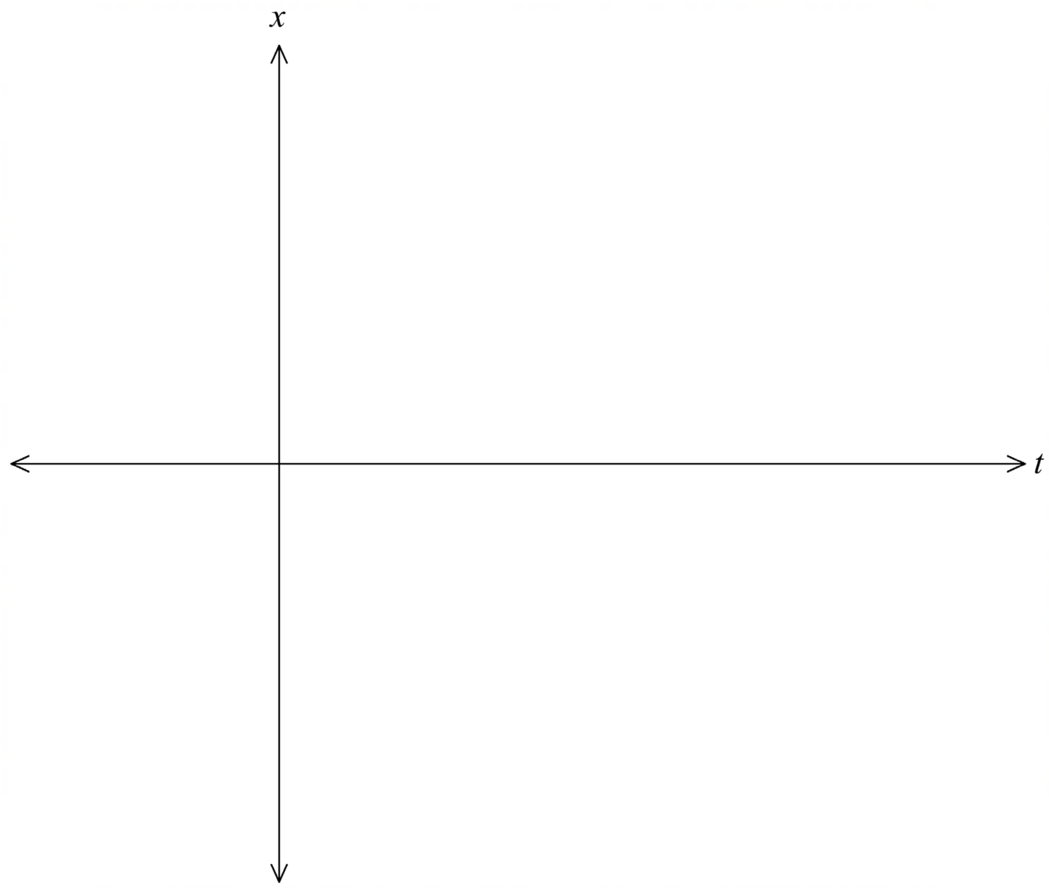
**Question 32 continues onto the next page**

e) Hence or otherwise, draw a neat sketch of $x = 4\cos\left(t - \frac{\pi}{3}\right)$ for $0 \le t \le \pi$, showing its main features. **3**




f) Calculate the total distance travelled between $t = \frac{\pi}{6}$ and $t = \frac{6\pi}{7}$. **3**

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**Question 33** (3 marks)

A radioactive substance is decaying such that its mass, $m$ grams, at a time $t$ years after initial observation is given by

$$m = 60e^{kt}\text{, where } k \text{ is a constant.}$$

After 100 years, the mass of radioactive substance is 42 grams.

a) Find the value of $k$, leaving your answer in exact form. **1**

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b) Find the value of $t$ when $m = 30$. **2**

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**Question 34** (5 marks)

\$2000 is invested into a credit union account at the beginning of each year and interest is paid at the end of each year, at a rate of 6.5% per annum, on the balance in the account at that time.

a) Show that the value of the investment at the end of the 3 years is \$6814.35. **2**

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b) At the end of how many years, before the next \$2000 is invested, would the accumulated amount in the account first exceed \$100,000? **3**

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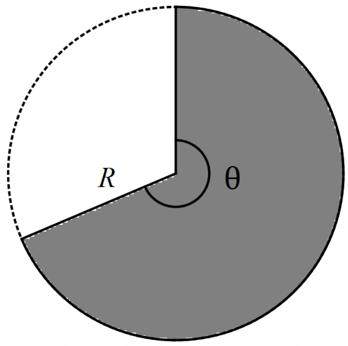
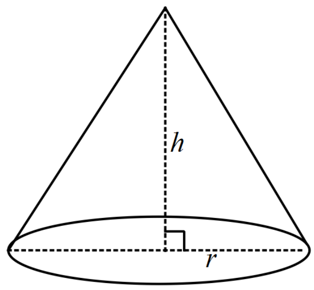
**Question 35** (9 marks)

From a circular disc of metal whose area is $100\text{m}^2$, a sector is cut out and used to make a right cone. The radius of the disc is $R$ metres. The perpendicular height of the cone with radius $r$ metres, is $h$ metres.




a) Show that the height of the cone is given by $h = \sqrt{\dfrac{100}{\pi} - r^2}$. **2**

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b) Show that the volume of the cone is given by $V = \dfrac{r^2\sqrt{100\pi - \pi^2 r^2}}{3}$. **1**

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**Question 35 continues onto the next page**

c) Show that the volume is maximised when $r = \sqrt{\dfrac{200}{3\pi}}$. **4**

**Question 35 continues onto the next page**

**Question 35 continues onto the next page**

d) Given that the area of the sector used to make the cone is $A = \pi Rr$, show that the angle in this sector $\theta$, which gives a maximum volume for the cone, is $\frac{2\pi\sqrt{6}}{3}$ radians. **2**

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**END OF PAPER**

# BAULKHAM HILLS HIGH SCHOOL

# YEAR 12 TRIAL EXAMINATION 2022

# MATHEMATICS ADVANCED

**NESA#**: ______________________________

**Teacher**: ______________________________

## Mathematics
## Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

*correct*

A ☒ B ☒ C ○ D ○

**Start Here** →

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | A ○ | B ○ | C ○ | D ○ | **6.** | A ○ | B ○ | C ○ | D ○ |
| **2.** | A ○ | B ○ | C ○ | D ○ | **7.** | A ○ | B ○ | C ○ | D ○ |
| **3.** | A ○ | B ○ | C ○ | D ○ | **8.** | A ○ | B ○ | C ○ | D ○ |
| **4.** | A ○ | B ○ | C ○ | D ○ | **9.** | A ○ | B ○ | C ○ | D ○ |
| **5.** | A ○ | B ○ | C ○ | D ○ | **10.** | A ○ | B ○ | C ○ | D ○ |

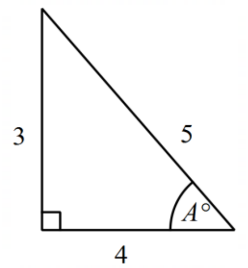
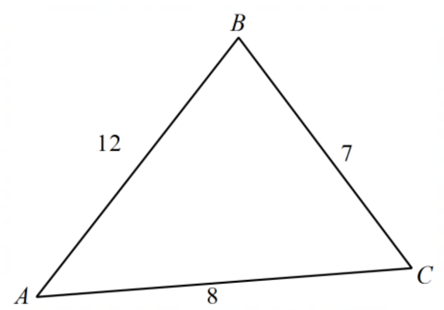
**Baulkham Hills High School**
**Task 4 Trials Examination 2022**

**Marking Guideline- Yr 12 Mathematics Advanced**

**Section I (10 marks)**

Award 1 marks to each correct answer.

Answers: 1.D 2.A 3.B 4.B 5. B 6.C 7.A 8.B 9.D 10.D

| Question | Suggested solutions | Answer |
|---|---|---|
| 1 | Line up 80% on the cumulative percentage to the ogive. This will show that 4 of the main causes takes up 80% of the problem. | D |
| 2 | 740g has a z-score of 2<br>$P(z>2)=50\%-47.5\%$<br>$=2.5\%$ | A |
| 3 | <br>$\cos A = \frac{3}{5}$<br>$\tan A = \frac{4}{3}$<br>$\therefore \cot A = \frac{3}{4}$ | B |
| 4 | $\cos A = \frac{8^2+12^2-7^2}{2(12)(8)}$<br>$=0.828125...$<br>$B=34°6'$(nearest minute)<br> | B |
| 5 | $2,b,c,d,1250$<br>The first term is 2.<br>The last term $1250=2\times r^4$<br>$r=5$<br>The GP is $2,10,50,250,1250$ | B |

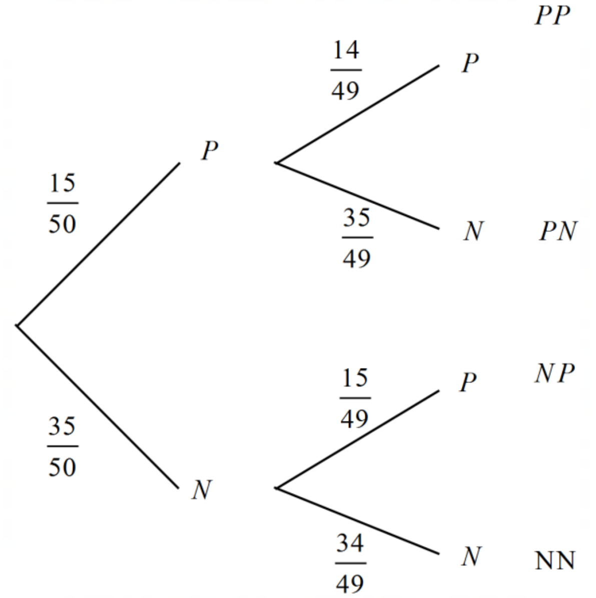
| | | |
|---|---|---|
| 6 | $\sum P(X=x)=1$<br>$\therefore 2b+b+4b+5b=1$<br>$\therefore b=\frac{1}{12}$<br>The expected mean of X:<br>$2b+2b+12b+20b=36b$<br>$=36\times\frac{1}{12}$<br>$=3$ | C |
| 7 | Looking at the denominator, $x$ cannot be equal to 4.<br>Domain: $(-\infty,4)\cup(4,\infty)$ | A |
| 8 | point A is the global minimum<br>point C is the local minimum<br>point D is the global maximum<br>$\therefore$ point B is the local maximum | B |
| 9 | To transform $y=\log_{10}x$ to $y=3\log_{10}(2(x+5))-4,$ the order is<br>1. Translate to the left by 5 units<br>2. Vertical dilation by a factor of 3<br>3. Horizontal dilation by a factor of $\frac{1}{2}$<br>4. Translate down by 4 units. | D |
| 10 | A probability of 61.79% has a $z$-score of 0.3<br>A probability of 35.94% has a $z$-score of -0.36<br>$z=\frac{\overline{x}-x}{\sigma}$<br>$0.3=\frac{22-\overline{x}}{\sigma}\quad -0.36=\frac{17-\overline{x}}{\sigma}$<br>Solving simultaneously:<br>$0.3\sigma=22-\overline{x}$<br>$-0.36=17-\overline{x}$<br>$\therefore \sigma=7.58$<br>$\therefore \overline{x}=19.73$ | D |

**Section II (90 marks)**

In all questions, award full marks for correct answers with necessary working.

Use the suggested solutions in conjunction with the marking criteria.

| Q | Suggested Solutions | Marking criteria |
|---|---|---|
| 11 | $2+\frac{t^2}{t+1}-\left(1+\frac{1}{t+1}\right)=\frac{2t+2+t^2-t-1-1}{t+1}$ <br> $=\frac{t^2+t}{t+1}$ <br> $=\frac{t(t+1)}{t+1}$ <br> $=t$ | 2- correction solution <br> 1- correctly making the denominator the same |
| 12 | $\text{LHS}=\tan\theta(1-\cot^2\theta)+\cot\theta(1-\tan^2\theta)$ <br> $=\frac{\tan\theta(\tan^2\theta-1)}{\tan^2\theta}+\frac{1}{\tan\theta}(1-\tan^2\theta)$ <br> $=\frac{\tan^2\theta-1+1-\tan^2\theta}{\tan\theta}$ <br> $=0$ <br> $=\text{RHS}$ | 1- correct solution |
| 13a | $\text{Period}=3\times 2$ <br> $=6\text{months}$ <br> $\text{Rate}=8\%\div 2$ <br> $=4\%$ <br> $\text{Future value}=700\times 6.6330$ <br> $=\$4643.10$ | 1- correct solution |
| 13b | $\text{Future value}=6000$ <br> $6000=x\times 6.6330$ <br> $x=\$904.57$ <br> <br> $\text{Angela needs}=\$904.57-\$700$ <br> $=\$204.57$ | 2- correct solution <br> 1- Obtaining $x=\$904.57$ |
| 14 | $y=\ln(4x-1)$ <br> $\frac{dy}{dx}=\frac{4}{4x-1}$ <br> $\frac{d^2y}{dx^2}=\frac{-16}{(4x-1)^2}$ | 2- correct solution <br> 1- correctly obtaining $\frac{dy}{dx}$ |
| 15 | $y=x^2e^{6x}$ <br> $y'=uv'+u'v$ <br> $y'=2xe^{6x}+6x^2e^{6x}$ <br> $u=x^2 \quad v=e^{6x}$ <br> $u'=2x \quad v'=6e^{6x}$ | 2- correct solution <br> 1- attempts to use the product rule |

| | | |
|---|---|---|
| 16 | $y=\dfrac{\sin 5x}{x^4}$<br>$y'=\dfrac{u'v-v'u}{v^2}$ $\quad u=\sin 5x \quad v=x^4$<br>$\quad u'=5\cos(5x) \quad v'=4x^3$<br>$y'=\dfrac{5x^4\cos(5x)-4x^3\sin(5x)}{x^8}$ | 2- correct solution<br>1- attempts to use the quotient rule |
| 17a |  | 2- Correct diagram<br>1- some significant progress<br>Note: they do not need to list the outcomes on the right for full marks |
| 17b | $\text{Probability(NP)}=\dfrac{15}{50}\times\dfrac{35}{49}$<br>$=\dfrac{3}{14}$ | 1- correct solution |
| 17c | $\text{Pr obability(at least 1 prize)}=1-(\text{no prize})$<br>$=1-\left(\dfrac{35}{50}\times\dfrac{34}{49}\right)$<br>$=\dfrac{18}{35}$ | 1- correct solution |
| 18 | $y=-x-1$<br>$m=-1$<br>$\theta=\tan^{-1}(-1)$<br>$\theta=-45°$<br>$\therefore \theta=135°$ | 2- correct solution<br>1- obtaining $m=-1$ |
| 19 | $f(x)'=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$<br>$=\lim\limits_{h\to 0}\dfrac{5(x+h)^2-3(x+h)-(5x^2-3x)}{h}$<br>$=\lim\limits_{h\to 0}\dfrac{5x^2+10xh+5h^2-3x-3h-5x^2+3x}{h}$<br>$=\lim\limits_{h\to 0}\dfrac{10xh+5h^2-3h}{h}$<br>$=\lim\limits_{h\to 0}\dfrac{h(10x+5h-3)}{h}$<br>$=\lim\limits_{h\to 0}10x+5h-3$<br>$=10x-3$ | 3- correct solution<br>2- correctly expands and simplifies<br>1- correctly substitutes $f(x+h)$ |

<table>
<tr><td>20</td><td>$\int_1^5 3e^{2x}dx = \frac{3}{2}\left[e^{2x}\right]_1^5$<br>$= \frac{3}{2}\left[e^{10} - e^2\right]$<br>$= 33028.6$ (1 decimal place)</td><td>2- correct solution<br>1- correctly integrates $3e^{2x}$<br>Note: Do not deduct rounding</td></tr>
<tr><td>21</td><td>$\log_5(x+1) - \log_5 x = 2$<br>$\log_5\left(\frac{x+1}{x}\right) = 2$<br>$5^2 = \frac{x+1}{x}$<br>$25x - x = 1$<br>$x = \frac{1}{24}$</td><td>2- correct solution<br>1- use the log law to solve the equation, or equivalent merit.</td></tr>
<tr><td>22</td><td>$\int_{\frac{\pi}{4}}^{\pi} \tan^2 x\,dx = \int_{\frac{\pi}{4}}^{\pi} (\sec^2 x - 1)\,dx$<br>$= \left[\tan x - x\right]_{\frac{\pi}{4}}^{\pi}$<br>$= \tan\pi - \pi - \tan\frac{\pi}{4} + \frac{\pi}{4}$<br>$= -\frac{3\pi}{4} - 1$</td><td>3- correct solution<br>2- Finds the anti-derivative of $\sec^2 x$, or equivalent merit<br>1- recognise the identity for $\tan^2 x$</td></tr>
<tr><td>23</td><td>$\int_1^3 \frac{dx}{5x-2}dx = \frac{2}{5}\int_1^3 \frac{5}{5x-2}dx$<br>$= \frac{2}{5}[\ln(5x-2)]_1^3$<br>$= \frac{2}{5}[\ln(5(3)-2) - \ln(5(1)-2))]$<br>$= \frac{2}{5}(\ln 13 - \ln 3)$<br>$= 0.59$ (2dp)</td><td>2- correct solution<br>1- Writes an anti-derivative involving the log function, or equivalent merit<br>Note: Do not deduct for incorrect rounding</td></tr>
<tr><td>24</td><td><table><tr><td>$x$</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>$y$</td><td>16</td><td>32</td><td>64</td><td>128</td></tr></table><br>$\int_4^7 2^x\,dx \approx \frac{1}{2}(16 + 128 + 2(32 + 64))$<br>$\approx 168$ (2dp)</td><td>3- correct solution with the correct table<br>2- correct table with some progress of using the trapezoidal rule<br>1- correct table or equivalent</td></tr>
</table>

<table>
<tr><td>25</td><td>$y = x\cos x$<br>$y' = \cos x - x\sin x$<br>When $x = \frac{\pi}{2}, y = 0$<br>$m_1 = \cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$<br>$m_1 = -\frac{\pi}{2}$<br>$m_2 = \frac{2}{\pi}$<br>$y - y_1 = m_2(x - x_1)$<br>$y - 0 = \frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$<br>$y = \frac{2x}{\pi} - 1$</td><td>3- correct solution<br>2- correctly obtains the normal gradient<br>1- correctly differentiates $y = x\cos x$</td></tr>
<tr><td>26a</td><td>$z = \frac{x - \bar{x}}{\sigma}$<br>$-2 = \frac{x - 8.1}{2}$<br>$x = 4.1\text{kg}$</td><td>1- correct solution</td></tr>
<tr><td>26b</td><td>$z = \frac{x - \bar{x}}{\sigma}$<br>$z = \frac{8.4 - 8.1}{2}$    $x = \frac{11 - 8.1}{2}$<br>$z = 0.15$    $z = 1.45$<br>$P(0.15 \le z \le 1.45) = P(z \le 1.45) - P(z \le 0.15)$<br>$= 0.9265 - 0.5596$<br>$= 0.3669$</td><td>3- correct solution<br>2- finds the probabilities for both z scores but does not subtract correctly<br>1- find the z score for both scores</td></tr>
<tr><td>27a</td><td>$f(x) = x^3 - 3x^2 + 3$<br>$f'(x) = 3x^2 - 6x$<br>$f''(x) = 6x - 6$<br>When $f'(x) = 0$<br>$0 = 3x^2 - 6x$<br>$0 = 3x(x - 2)$<br>$x = 0, 2$<br>When $x = 0, y = 3$    When $x = 2, y = -1$<br>$f''(0) = -6$    $f''(2) = 6$<br>$< 0 \therefore$ maximum turning point    $> 0 \therefore$ minimum turning point</td><td>3- correct solution<br>2- correctly obtains $x = 0, 2$<br>1- some progress</td></tr>
<tr><td>27b</td><td>When $f''(x) = 0$<br>$6x - 6 = 0$<br>$x = 1$<br><br>Check concavity changes<table><tr><td>$x$</td><td>0.5</td><td>1</td><td>1.5</td></tr><tr><td>$f''(x)$</td><td>$-3$</td><td>0</td><td>3</td></tr></table>$\therefore$ (1,1) is a point of inflection</td><td>2- finds the coordinate and checks the concavity<br>1- finds (1,1)</td></tr>
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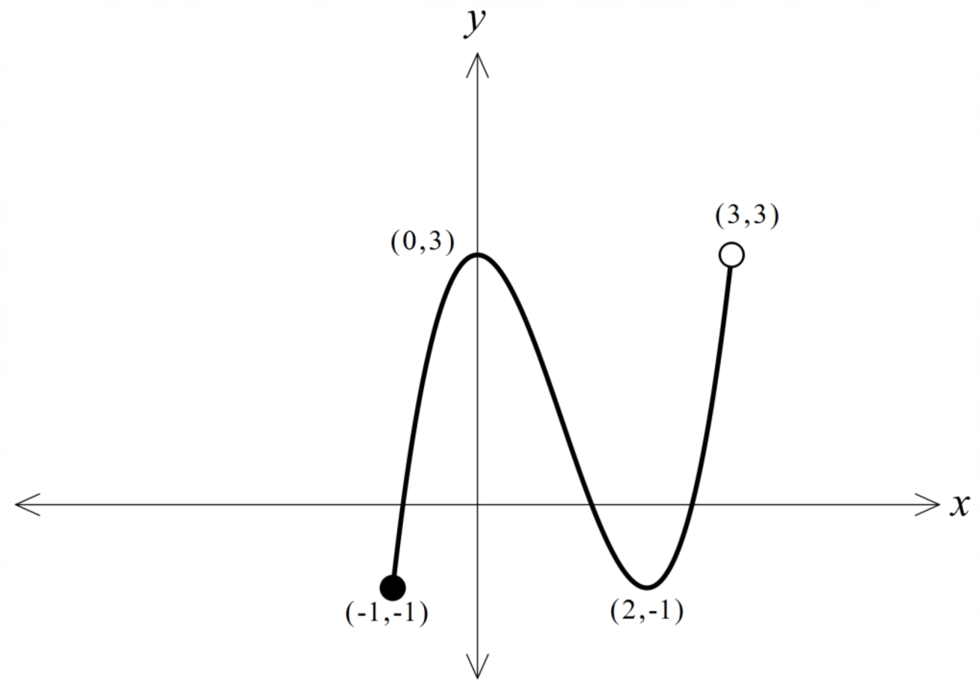
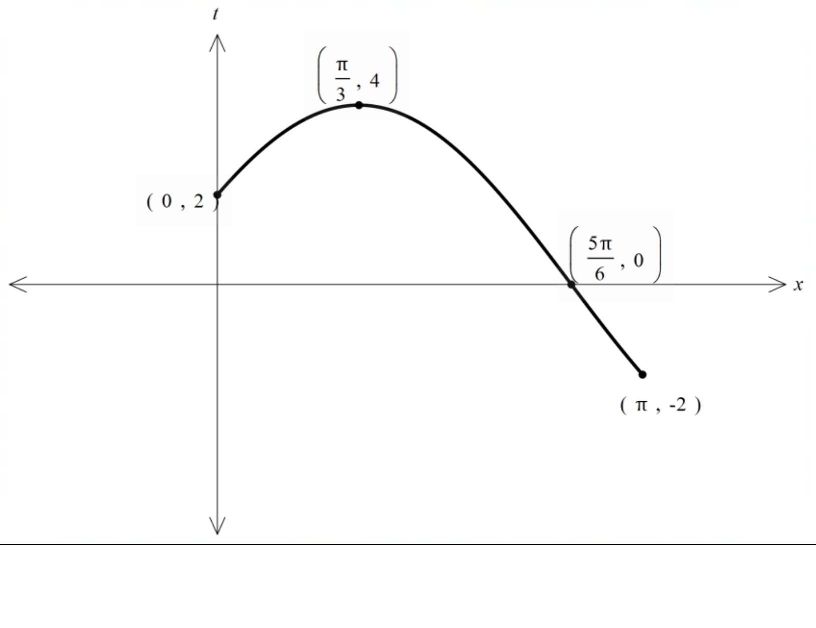
| 27c |  | 3-correct graph showing all features including the endpoints<br>2- correct graph with all features but not including the endpoints<br>1- correct shape |
| 28 | $2^{2x}-2^2(2)^x=32$<br>Let $u=2^x$<br>$u^2-4u-32=0$<br>$(u-8)(u+4)=0$<br>$u=8 \quad u=-4$<br>$8=2^x \quad -4=2^x$<br>$x=3 \quad \therefore$ no solution<br>$\therefore x=3$ | 3- correct solution<br>2- correctly obtains $u=8$ or $-4$<br>1- correctly reduces the exponential to a quadratic equation and factorises. |
| 29 | $\int_0^{\frac{\pi}{6}}(\sin x-(1-\cos(2x)))\,dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(1-\cos(2x)-\sin x)\,dx$<br>$=\left[-\cos x-x+\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}}+\left[x-\frac{1}{2}\sin 2x+\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$<br>$=\left(-\cos\left(\frac{\pi}{6}\right)-\frac{\pi}{6}+\frac{1}{2}\sin 2\left(\frac{\pi}{6}\right)+\cos 0-0+\frac{1}{2}\sin 0\right)$<br>$+\left(\frac{\pi}{2}-\frac{1}{2}\sin 2\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{2}\right)+-\frac{\pi}{6}\frac{1}{2}\sin\left(2\times\frac{\pi}{6}\right)-\cos\frac{\pi}{6}\right)$<br>$=0.66$ (2dp) | 3- correct solution<br>2- obtains correct primitive or equivalent merit<br>1- obtains correct definite integral, or equivalent merit |

| | | |
|---|---|---|
| 30a | $\int_0^k 3^x = 1$<br>$\left[\frac{3^x}{\ln 3}\right]_0^k = 1$<br>$\frac{3^k}{\ln 3} - \frac{3^0}{\ln 3} = 1$<br>$3^k = \ln 3 + 1$<br>$k = \log_3(\ln(3)+1))$<br>$= 0.67(2\text{dp})$ | 2- correct solution<br>1- Writes an equation involving a definite integral set equal to 1, or equivalent merit |
| 30b | $F(x) = 0.8$<br>$F(x) = \int 3^x$<br>$= \frac{3^x}{\ln 3} + C$<br>When $x = 0, F(0) = 0$<br>$0 = \frac{3^0}{\ln 3} + C$<br>$C = -\frac{1}{\ln 3}$<br>$F(x) = \begin{cases} 0, & \text{for } x \le 0 \\ \frac{3^x - 1}{\ln 3}, & \text{for } 0 \le x \le 0.67 \\ 1, & \text{for } x > 0.67 \end{cases}$<br>$\frac{3^x - 1}{\ln 3} = 0.8$<br>$3^x = 0.8\ln(3) + 1$<br>$x = \log 3(0.8\ln(3)+1)$<br>$= 0.57 \text{ (2dp)}$ | 2- correct solution<br>1- correctly identifies<br>$0.8 = \frac{3^x - 1}{\ln 3}$ |
| 31 | $f''(t) = t + t^{\frac{1}{2}}$<br>$f'(t) = \frac{t^2}{2} + \frac{2}{3}t^{\frac{2}{3}} + C,$ where C is some constant<br>$2 = \frac{1}{2} + \frac{2}{3} + C$<br>$\therefore C = \frac{5}{6}$<br>$f'(t) = \frac{t^2}{2} + \frac{2}{3}t^{\frac{3}{2}} + \frac{5}{6}$<br>$f(t) = \frac{t^3}{6} + \frac{4}{15}t^{\frac{5}{2}} + \frac{5}{6}t + D,$ where D is some constant<br>$1 = \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D$<br>$\therefore D = -\frac{4}{15}$<br>$\therefore f(t) = \frac{t^3}{6} + \frac{4}{15}t^{\frac{5}{2}} + \frac{5}{6}t - \frac{4}{15}$ | 3- correct solution<br>2- obtains correct primitive of $f'(x)$ or equivalent merit<br>1- obtains correct primitive of $f''(x)$ or equivalent merit |

| | | |
|---|---|---|
| 32a | When $t=0, x=4\cos\left(-\frac{\pi}{3}\right)$<br>$\therefore x=2$ | 1- correct solution |
| 32b | Particle at rest when $v=0$<br>$v=-4\sin\left(t-\frac{\pi}{3}\right)$<br>$0=-4\sin\left(t-\frac{\pi}{3}\right)$<br>$0=\sin\left(t-\frac{\pi}{3}\right)$<br>$t-\frac{\pi}{3}=0$<br>$t=\frac{\pi}{3}$<br>$\therefore$ The particle first comes to rest is when $t=\frac{\pi}{3}$ | 2- correct solution<br>1- differentiates correctly to find $v$ |
| 32c | $x=4\cos\left(t-\frac{\pi}{3}\right)$<br>$0=4\cos\left(t-\frac{\pi}{3}\right)$<br>$t-\frac{\pi}{3}=\frac{\pi}{2}$<br>$\therefore t=\frac{5\pi}{6}$ | 1- correct solution |
| 32d | $a=-4\cos\left(t-\frac{\pi}{3}\right)$<br>$0=-4\cos\left(t-\frac{\pi}{3}\right)$<br>$0=\cos\left(t-\frac{\pi}{3}\right)$<br>$t-\frac{\pi}{3}=\frac{\pi}{2}$<br>$\therefore t=\frac{5\pi}{6}$ | 2- correct solution<br>1- differentiates correctly to find $a$ and substitutes $a=0$ |
| 32e |  | 3- correct graph showing all features<br>2- correct graph but missing some features<br>1- incorrect graph showing all features |

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| 32f | When $t=\frac{\pi}{6}, x=4\cos\left(-\frac{\pi}{6}\right)$<br>When $t=\frac{6\pi}{7}, x=4\cos\left(\frac{11\pi}{21}\right)$<br>Total distance $=\left(4-4\cos\left(-\frac{\pi}{6}\right)\right)+4+\left\|4\cos\left(\frac{11\pi}{21}\right)\right\|$<br>$=4.835(3\text{dp})$ | 3- correct solution<br>2- finding either $f\left(\frac{\pi}{6}\right)$ or $f\left(\frac{6\pi}{7}\right)$ with $+4$<br>1- realising to separate the distances |
| 33a | $m=60e^{kt}$<br>$42=60e^{100k}$<br>$0.7=e^{100k}$<br>$k=\frac{1}{100}\ln 0.7$ | 1- correct solution |
| 33b | $m=60e^{\frac{t}{100}\ln 0.7}$<br>$30=60e^{\frac{t}{100}\ln 0.7}$<br>$0.5=e^{\frac{t}{100}\ln 0.7}$<br>$\ln 0.5=\frac{t}{100}\ln 0.7$<br>$t=100\times\frac{\ln 0.5}{\ln 0.7}$<br>$t=194.34(2\text{dp})$ | 2- correct solution<br>1- correctly obtaining $\ln 0.5=\frac{t}{100}\ln 0.7$ or equivalent merit |
| 34a | Value of investment at the end of 3 years<br>$=2000(1.065)^3+2000(1.065)^2+2000(1.065)$<br>$=\$6814.35$ | 2- correct solution<br>1- correctly obtaining the series |
| 34b | Amount accumulated at end of $n^{th}$ year<br>$=2000\times1.065^n+2000\times1.065^{n-1}+\ldots+2000\times1.065$<br>$=2000(1.065+1.065^2+\ldots+1.065^n)$<br>$=2000\times\frac{1.605(1.065^n-1)}{1.065-1}$<br>$\frac{2130}{0.065}(1.065^n-1)>100\,000$<br>$\frac{2130}{0.065}(1.065^n-1)>100\,000$<br>$1.065^n>4.0516$<br>$n>\frac{\log 4.0516}{\log 1.065}$<br>$n>22.2$<br>$\therefore$ Amount first exceeds \$100 000 at the end of 23 years. | 3- correct solution<br>2- identifying the sum of a GP correctly<br>1- correctly finding the series that shows the amount accumulated at the end of the nth year. |

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| 35a | Area: $\pi R^2 = 100$<br>$\therefore R^2 = \frac{100}{\pi}$<br>Using pythagoras theorem<br>$R^2 = h^2 + r^2$<br>$h = \sqrt{R^2 - r^2}$<br>$\therefore h = \sqrt{\frac{100}{\pi} - r^2}$ | 2- Correct solution<br>1- correctly substituting $R = \sqrt{\frac{100}{\pi}}$ |
| 35b | $V = \frac{1}{3}\pi r^2 h$<br>$= \frac{1}{3}\pi r^2 \sqrt{\frac{100}{\pi} - r^2}$<br>$= \frac{1}{3}\pi r^2 \sqrt{\frac{100 - \pi r^2}{\pi}}$<br>$= \frac{1}{3} r^2 \sqrt{\pi^2 \times \frac{100 - \pi r^2}{\pi}}$<br>$= \frac{r^2\sqrt{100\pi - \pi^2 r^2}}{3}$ | 1- correct solution |
| 35c | $v = \sqrt{\frac{r^4(100\pi - \pi^2 r^2)}{9}}$<br>$= \left(\frac{100\pi r^4}{9} - \frac{\pi^2 r^6}{9}\right)^{\frac{1}{2}}$<br>$v' = \frac{1}{2}\left(\frac{100\pi r^4}{9} - \frac{\pi^2 r^6}{9}\right)^{-\frac{1}{2}} \times \left(\frac{400\pi r^3}{9} - \frac{6\pi^2 r^5}{9}\right)$<br>$= \frac{\left(\frac{400\pi r^3}{9} - \frac{6\pi^2 r}{9}\right)}{2\sqrt{\frac{100\pi r^4}{9} - \frac{\pi^2 r^6}{9}}}$<br>$v' = 0$ when $\frac{400\pi r^3}{9} - \frac{6\pi^2 r^5}{9} = 0$<br>$\pi r^3(400 - 6\pi r^2) = 0$<br>$400 - 6\pi r^2 = 0$<br>$r = 0, \quad r = \sqrt{\frac{200}{3\pi}}$<br>Test for max when $r = \sqrt{\frac{200}{3\pi}}$<br>$v' = \frac{\left(\frac{400\pi r^3}{9} - \frac{6\pi^2 r}{9}\right)}{2\sqrt{\frac{100\pi r^4}{9} - \frac{\pi^2 r^6}{9}}}$ | 4- correct solution<br>3- correctly obtained $r = \sqrt{\frac{200}{3\pi}}$ but did not check it was a maximum<br>2- some significant progress towards obtaining $r$<br>1- correctly finds $v'$ in any form |

| | | | |
|---|---|---|---|
| $r$ | 4 | $\sqrt{\frac{200}{3\pi}}$ | 5 |
| $\frac{dv}{dr}$ | 16.49 | 0 | $-22.71$ |
| | positive | 0 | negative |

<table>
<tr>
<td>35d</td>
<td>
$A = \pi R r$<br>
area of a sector $A = \frac{\theta}{2} \times R^2$<br>
$\therefore \frac{\theta}{2} \times R^2 = \pi R r$<br>
$\frac{\theta}{2}\sqrt{\frac{100}{\pi}} = \pi r$<br>
$\theta = \frac{2\pi r}{\sqrt{\frac{100}{\pi}}}$<br>
$\theta = \frac{2\pi\sqrt{\frac{200}{3\pi}}}{\sqrt{\frac{100}{\pi}}}$<br>
$= 2\pi\sqrt{\frac{200}{3\pi} \times \frac{\pi}{100}}$<br>
$= 2\pi \times \sqrt{\frac{2}{3}}$<br>
$= 2\pi \times \sqrt{\frac{2}{3} \times \frac{3}{3}}$<br>
$= 2\pi \times \sqrt{\frac{6}{9}}$<br>
$\therefore \theta = \frac{2\pi\sqrt{6}}{3}$ radians
</td>
<td>2- correct solution<br>1-using simultaneous equations, substituting all the information provided from the previous parts</td>
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